



First ADS analysis of $B^- \rightarrow D^0 K^-$ decays in hadron collisions

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for the CDF collaboration

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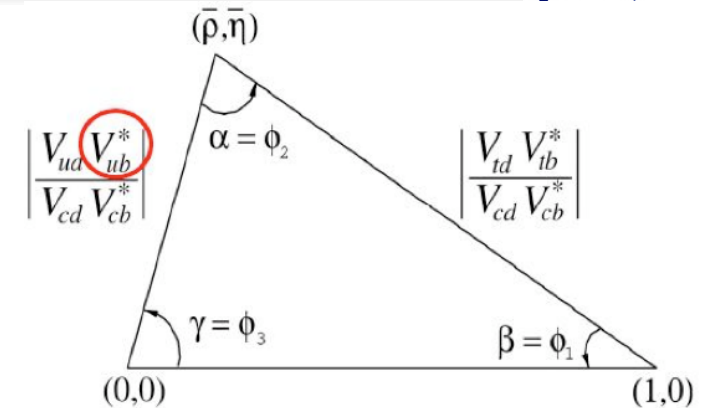
Motivation: CKM γ angle measurement

CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

CP violation if $\eta \neq 0$

$b \rightarrow u$ transition
B meson system



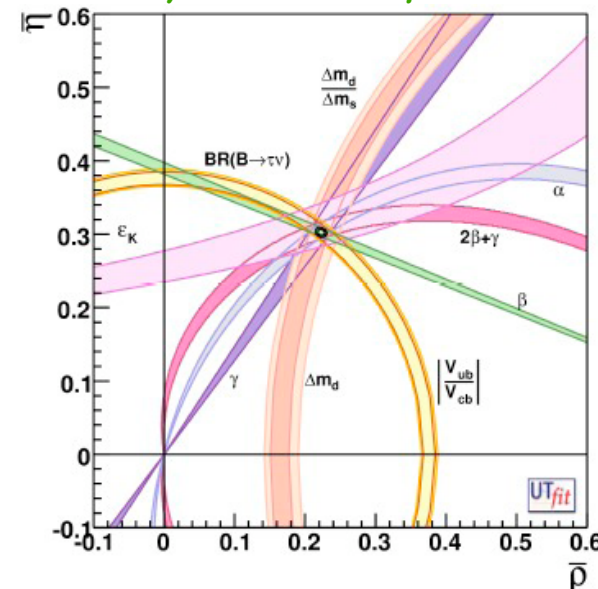
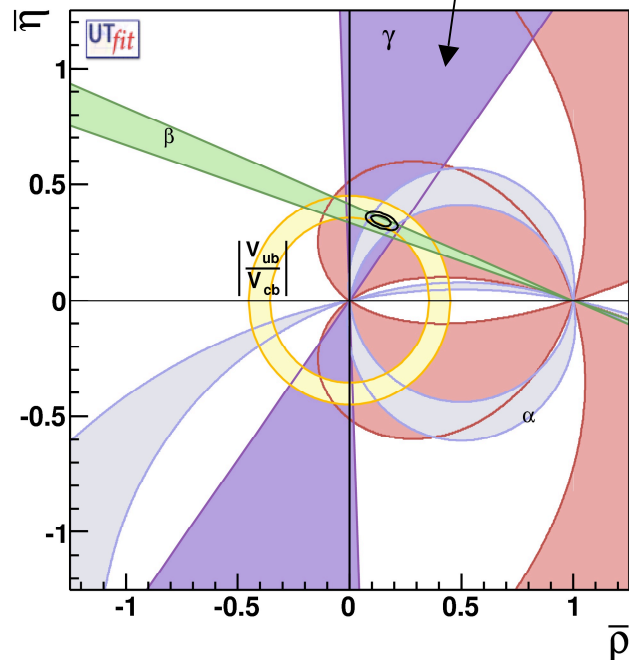
γ is the least well-known angle of the CKM triangle nowadays

Can probe **New Physics**

(select ρ - η values valid in most of the NP extensions)

Improving the resolution can lead to:

- *consistency with SM prediction*

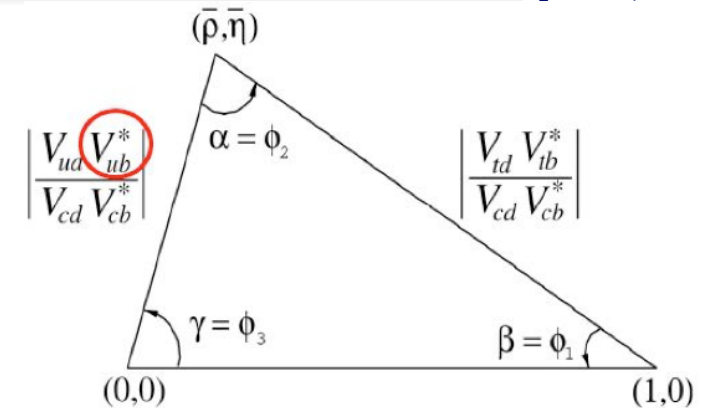


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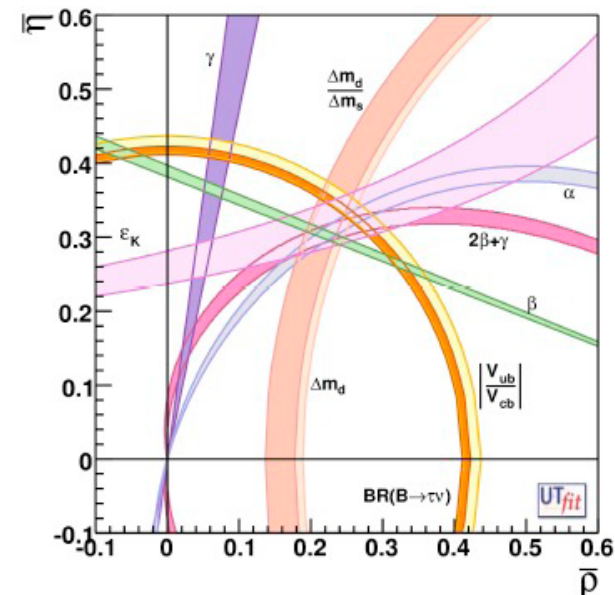
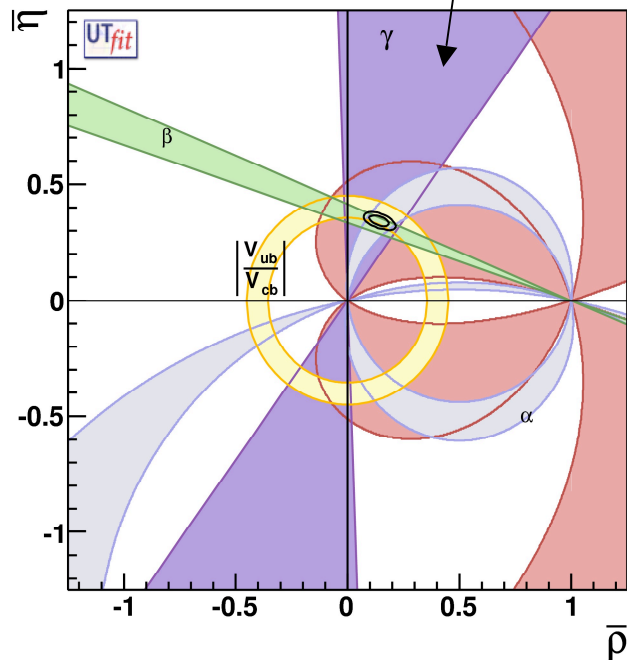
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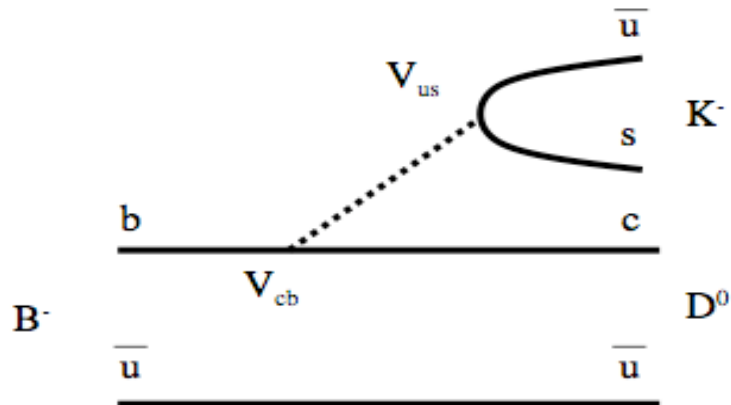
- **NP scenario**



Use of $B \rightarrow DK$ decays is the cleanest way to measure γ :

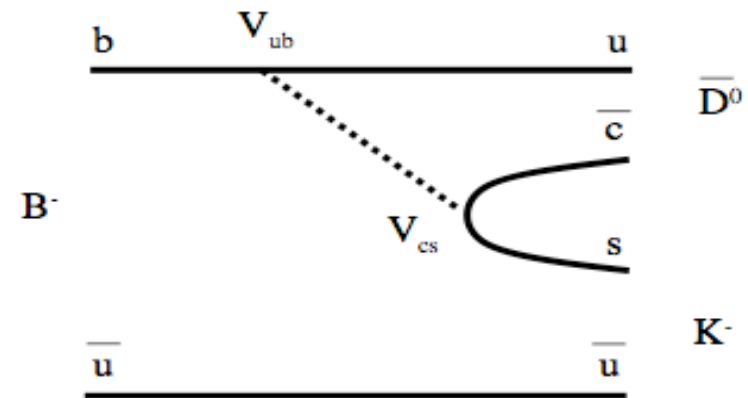
- tree-level amplitude only
- tiny theoretical uncertainties

γ can be extracted exploiting the **interference** between the processes $b \rightarrow c \bar{u} s$ ($B^- \rightarrow D^0 K^-$) and $b \rightarrow u \bar{c} s$ ($B^- \rightarrow \bar{D}^0 K^-$), when D^0 and \bar{D}^0 decay to the same final state



Favored $b \rightarrow c$ transition

$$A_1 \sim V_{cb} V_{us}^* \sim \lambda^3$$

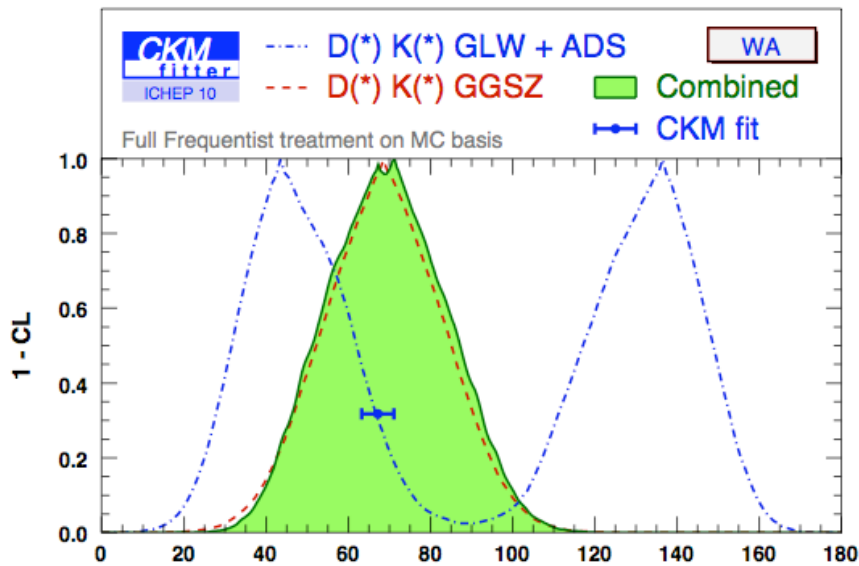


Color suppressed $b \rightarrow u$ transition

$$A_2 \sim V_{ub} V_{cs}^* \sim \lambda^3 r_B e^{-i\delta_B} e^{-i\gamma}$$

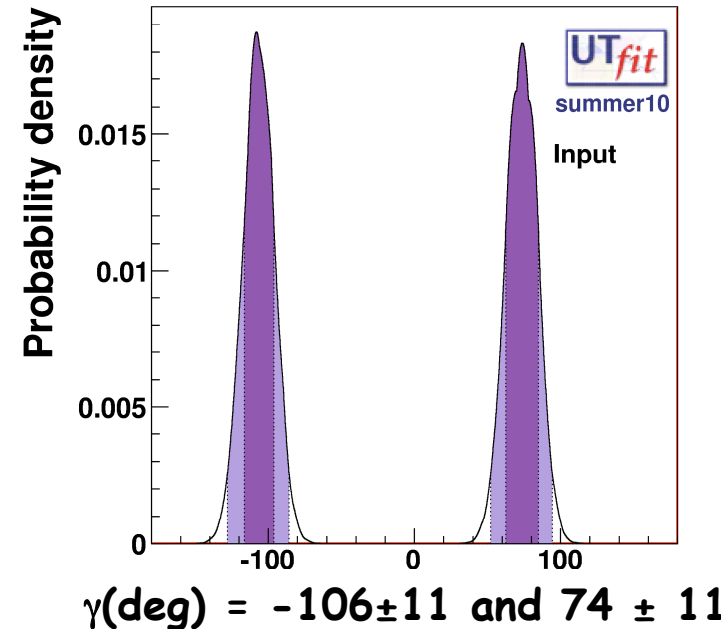


Current situation for the γ angle measurement using $B^- \rightarrow D^0 K^-$



$$\gamma(\text{deg}) = 71 [+21 \ -25]$$

Still large statistical uncertainty



$$\gamma(\text{deg}) = -106 \pm 11 \text{ and } 74 \pm 11$$

- **GGSZ (Giri-Grossmann-Soffer-Zupan) method** ([PRL78,3257, PRD68,054018])

that uses the $B^\pm \rightarrow D K^\pm$ decays with the D^0 and \bar{D}^0 reconstructed into three-body final state. For example the $D^0 \rightarrow K^0_s \pi^+ \pi^-$

- **GLW (Gronau-London-Wyler) method** ([PLB253,483 PLB265,172])

that uses the $B^\pm \rightarrow D K^\pm$ decays with D_{CP} decay modes. $D_{CP+} \rightarrow \pi^+ \pi^-, K^+ K^-$ and $D_{CP-} \rightarrow K^0_s \pi^0, K^0_s \omega, K^0_s \phi$.

- **ADS (Atwood-Dunietz-Soni) method** ([PRL78,3257;PRD63,036005])

that uses the $B^\pm \rightarrow D K^\pm$ decays with D reconstructed in the doubly Cabibbo suppressed $D^0_{DCS} \rightarrow K^+ \pi^-$



ADS method



ADS Observables



Direct CP violation in $B \rightarrow D_{DCS}K$ modes

- expected large CP asymmetry
- decay suppressed by a factor of $\sim 10^{-3}$ wrt favored
- results have to be combined with other methods to obtain γ measurement



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Observables

$$R_{ADS}(K) = \frac{N(B^- \rightarrow D_{DCS}^0 K^-) + N(B^+ \rightarrow D_{DCS}^0 K^+)}{N(B^- \rightarrow D_{CF}^0 K^-) + N(B^+ \rightarrow D_{CF}^0 K^+)}$$

$$\mathcal{A}_{ADS}(K) = \frac{N(B^- \rightarrow D_{DCS}^0 K^-) - N(B^+ \rightarrow D_{DCS}^0 K^+)}{N(B^- \rightarrow D_{DCS}^0 K^-) + N(B^+ \rightarrow D_{DCS}^0 K^+)}$$

$$D_{CF}^0 \rightarrow K^- \pi^+, D_{DCS}^0 \rightarrow K^+ \pi^-$$

From theory:

$$R_{ADS}(K) = r_D^2 + r_B^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \gamma$$

$$\mathcal{A}_{ADS}(K) = 2r_B r_D \sin(\delta_B + \delta_D) \sin \gamma / R_{ADS}(K)$$



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$$D_{CF}^0 \rightarrow K^- \pi^+, D_{DCS}^0 \rightarrow K^+ \pi^-$$

$$\mathcal{A}_{ADS}(MAX) = \frac{2r_B r_D}{r_B^2 + r_D^2}$$

Sizeable asymmetries may be found also for $B \rightarrow D_{DCS} \pi$

$$R_{ADS}(\pi) = \frac{N(B^- \rightarrow D_{DCS}^0 \pi^-) + N(B^+ \rightarrow D_{DCS}^0 \pi^+)}{N(B^- \rightarrow D_{CF}^0 \pi^-) + N(B^+ \rightarrow D_{CF}^0 \pi^+)}$$

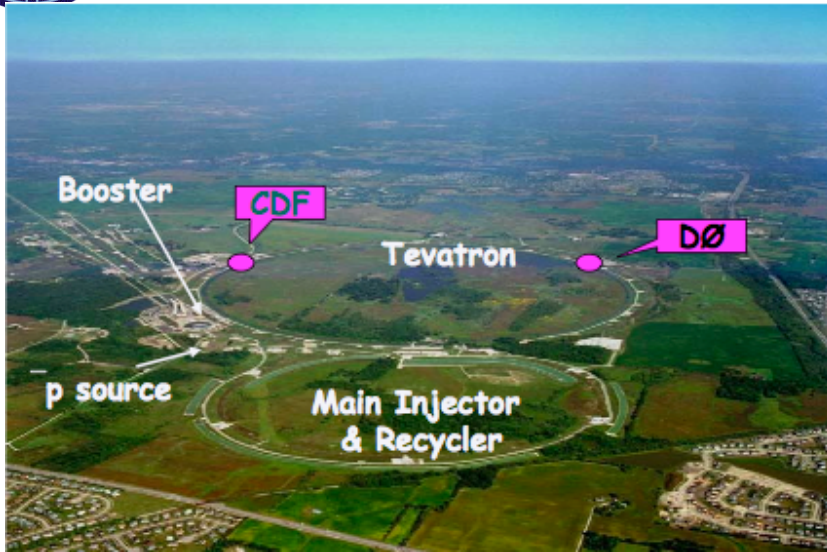
$$\mathcal{A}_{ADS}(\pi) = \frac{N(B^- \rightarrow D_{DCS}^0 \pi^-) - N(B^+ \rightarrow D_{DCS}^0 \pi^+)}{N(B^- \rightarrow D_{DCS}^0 \pi^-) + N(B^+ \rightarrow D_{DCS}^0 \pi^+)}$$

ADS method at CDF

First measurement of A_{ADS} and R_{ADS}
at a hadron collider

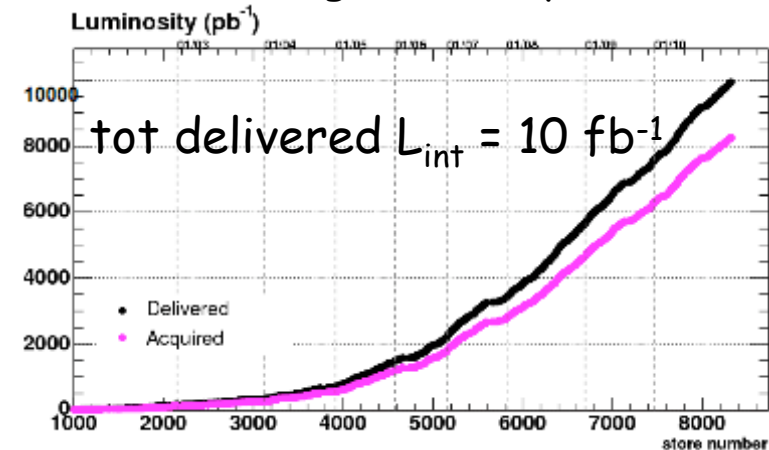


The Tevatron



Good performances on Run II:

- peak $L_{\text{inst}} = 3.5\text{--}4 \cdot 10^{32} \text{cm}^{-2}\text{s}^{-1}$
- delivering $2.5 \text{fb}^{-1}/\text{year}$

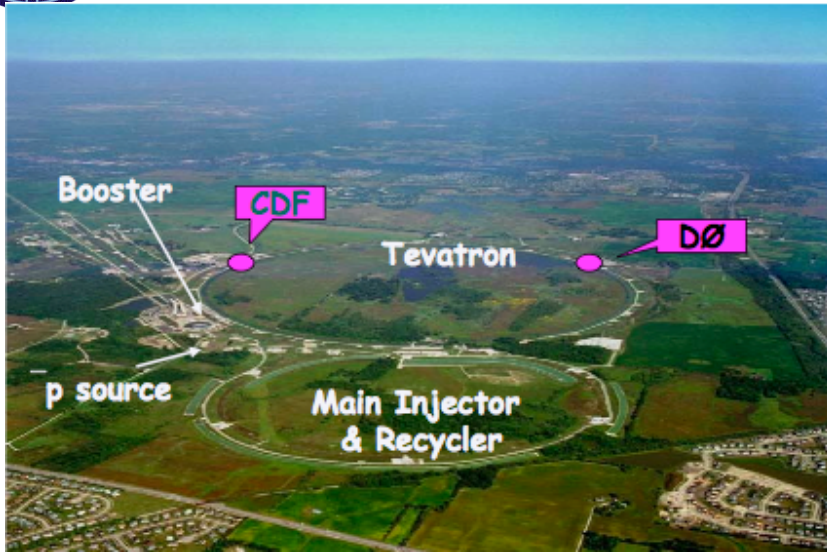


Tevatron is great for rare B decay searches:

- **Enormous b production cross section**
(x1000 times larger than e^+e^- B factories)
- **All B species** are produced (B^0 , B^+ , B_s , Λ_b ...)

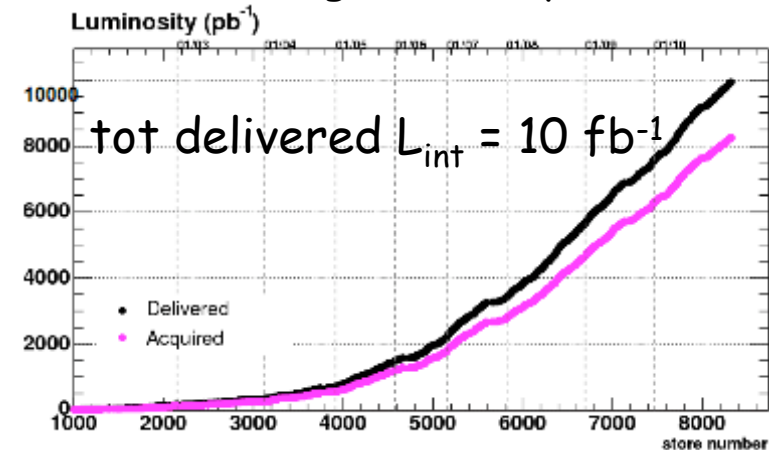


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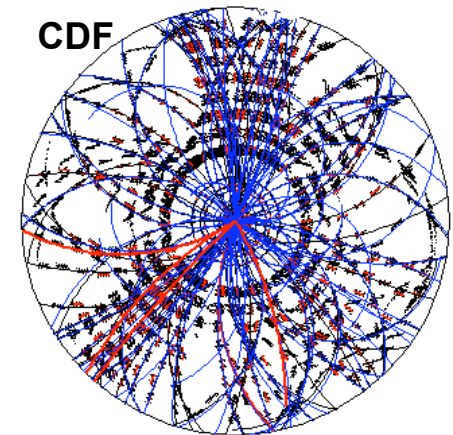
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But:

- The **total inelastic x-section** is a factor 10^3 larger than $\sigma(b\bar{b})$
- The **BRs** of rare b-hadron decays are $\mathcal{O}(10^{-6})$ or lower

Interesting events must be extracted from a high track multiplicity environment



Detectors need to have:

- Very good tracking and vertex resolution and highly selective trigger



The CDF II detector



TRACKING system:

- **DRIFT CHAMBER**

96 layers ($|\eta| < 1$)

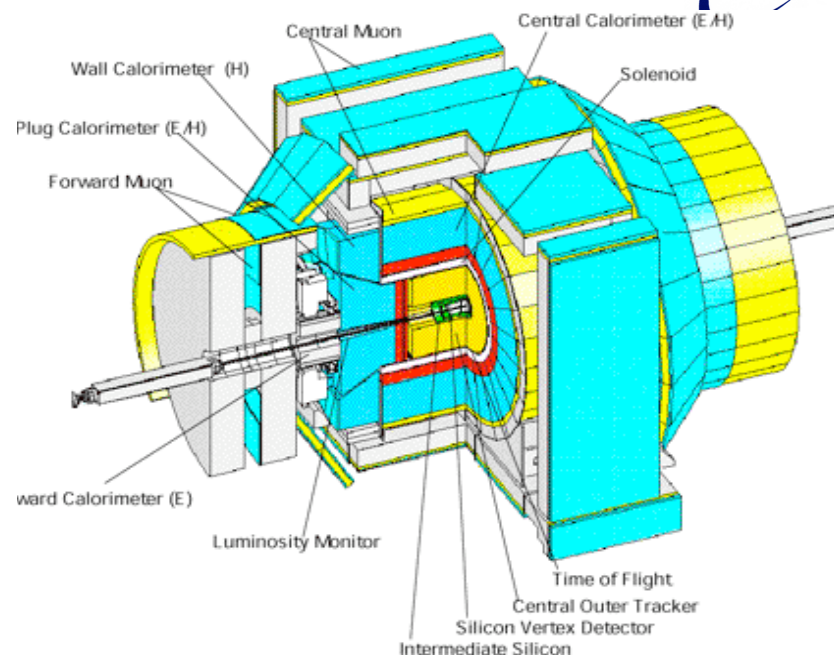
→ 1.5σ π/K separation by dE/dx

- **SILICON TRACKER**

7 layers (1.5-22cm from beam pipe)

→ I.P. resolution $35 \mu\text{m}$ at 2 GeV

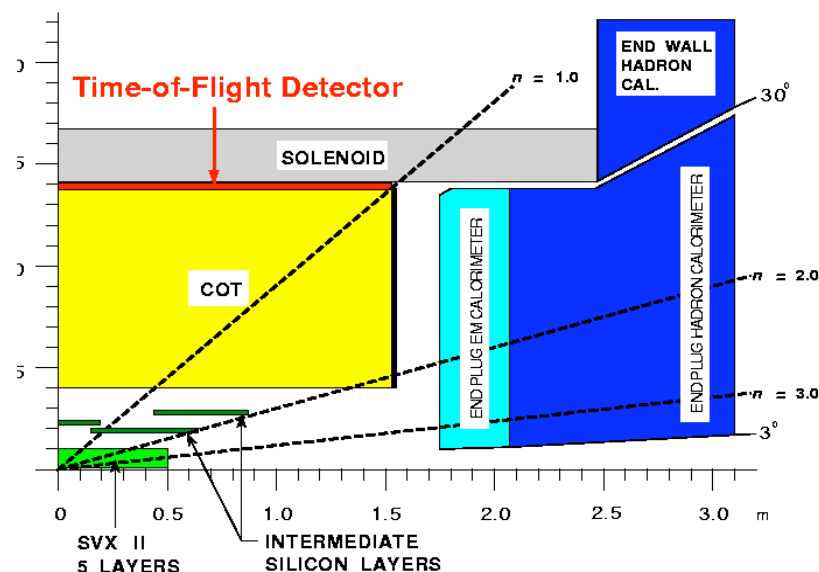
→ $\sigma(p_T)/p_T^2 \sim 0.015\%$ (c/GeV)



TRACKING TRIGGER system:

- **Chamber track processor at L1,**
2D tracks in COT, $p_T > 1.5 \text{ GeV}$

- **Silicon Vertex Trigger at L2,**
2D tracks $p_T > 2 \text{ GeV}$,
Impact Parameter measurement (trigger
on events containing long lived particles)



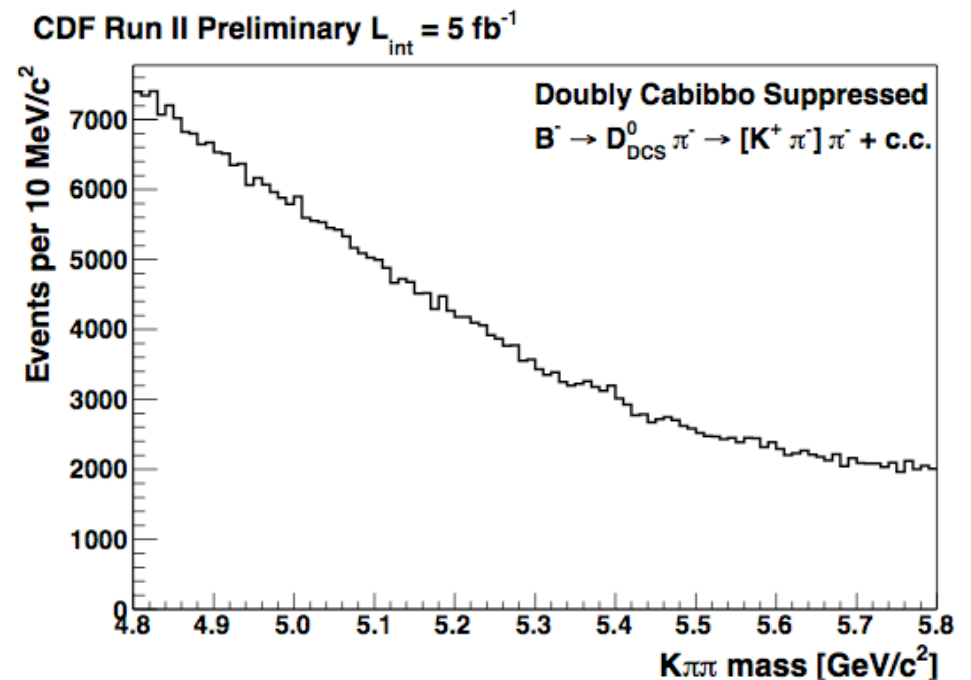
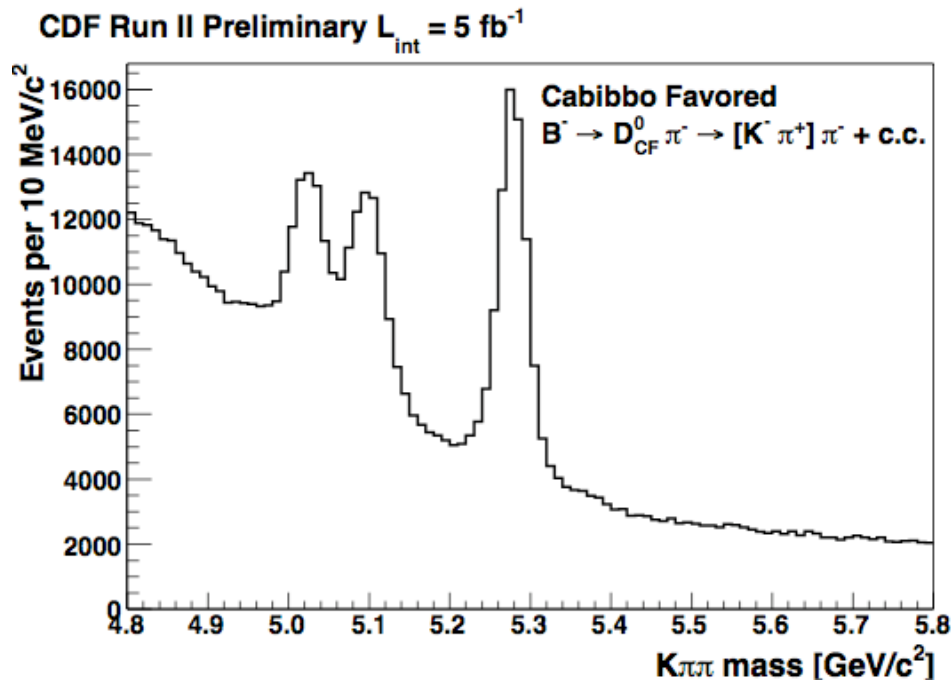


CF and DCS samples ($L = 5\text{fb}^{-1}$)



$$B^- \rightarrow D_{\text{CF}}^0 \pi^- \rightarrow [K^- \pi^+] \pi^-$$

$$B^- \rightarrow D_{\text{DCS}}^0 \pi^- \rightarrow [K^+ \pi^-] \pi^-$$



Cuts optimization



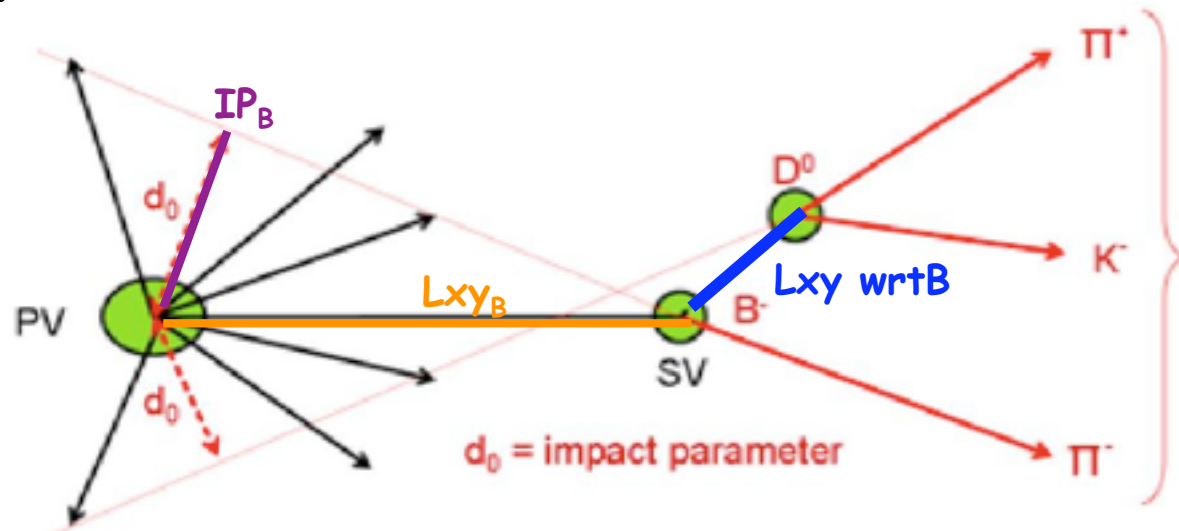
Crucial step toward
the DCS modes

- Maximize the quantity $\frac{S}{1.5 + \sqrt{B}}$ on CF sample. (arXiv:0808063v2)

D^0 candidate

Cuts on:

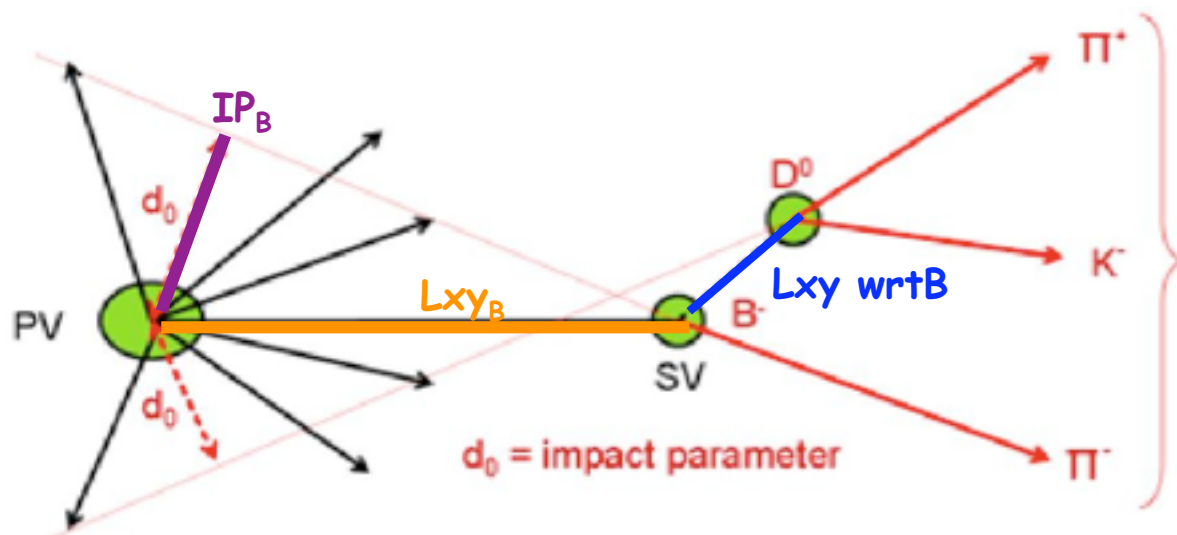
- the invariant mass
- angular distribution
- the decay length wrt B to remove $B \rightarrow 3$ body decays
- particle identification of tracks from D^0 to remove $D^0 \rightarrow \pi\pi$ events



B candidate

Cuts on:

- decay length wrt primary vertex
- impact parameter
- angle between momentum and decay length



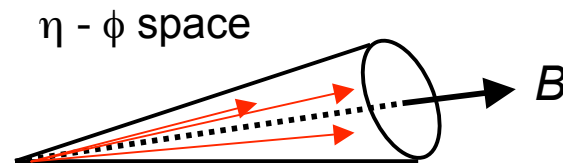
B candidate

Cuts on:

- decay length wrt primary vertex
- impact parameter
- angle between momentum and decay length

- **isolation**

$$I(B) = \frac{p_T(B)}{p_T(B) + \sum_i p_T(i)}$$



- **3D vertex quality**, obtained with the 3D silicon-tracking, to:
 - resolve multiple vertices along the beam direction
 - reject fake tracks.

Backg. reduces x2, small inefficiency on signal (<10%).

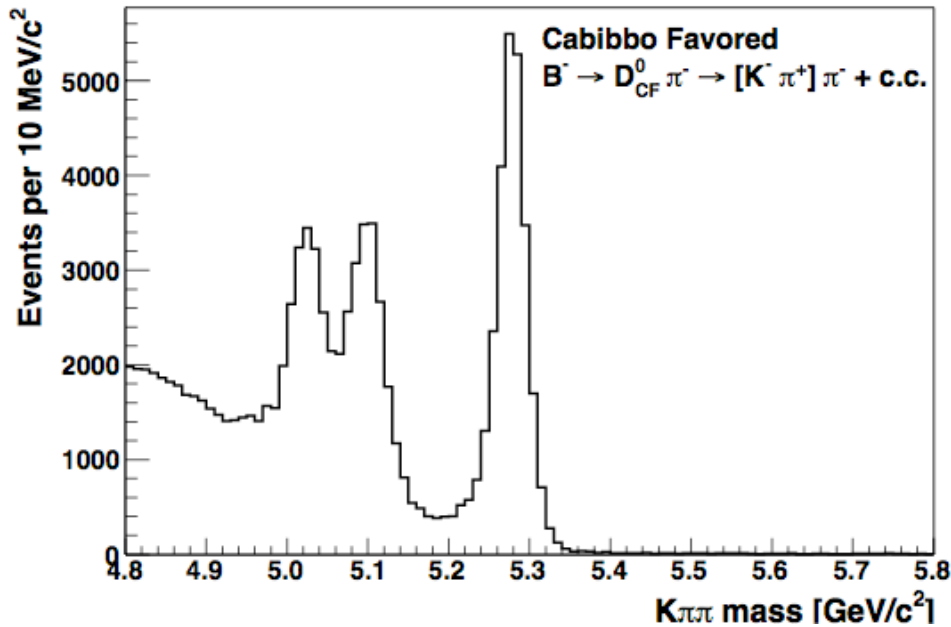


CF and DCS after cut optimization



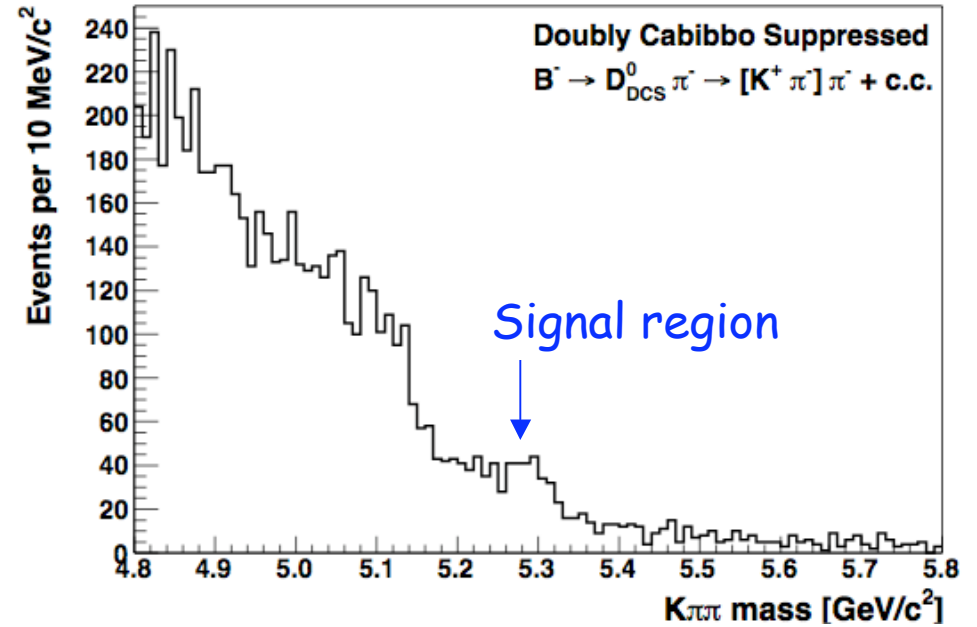
$$B^- \rightarrow D_{CF}^0 \pi^- \rightarrow [K^- \pi^+] \pi^-$$

CDF Run II Preliminary $L_{int} = 5 \text{ fb}^{-1}$



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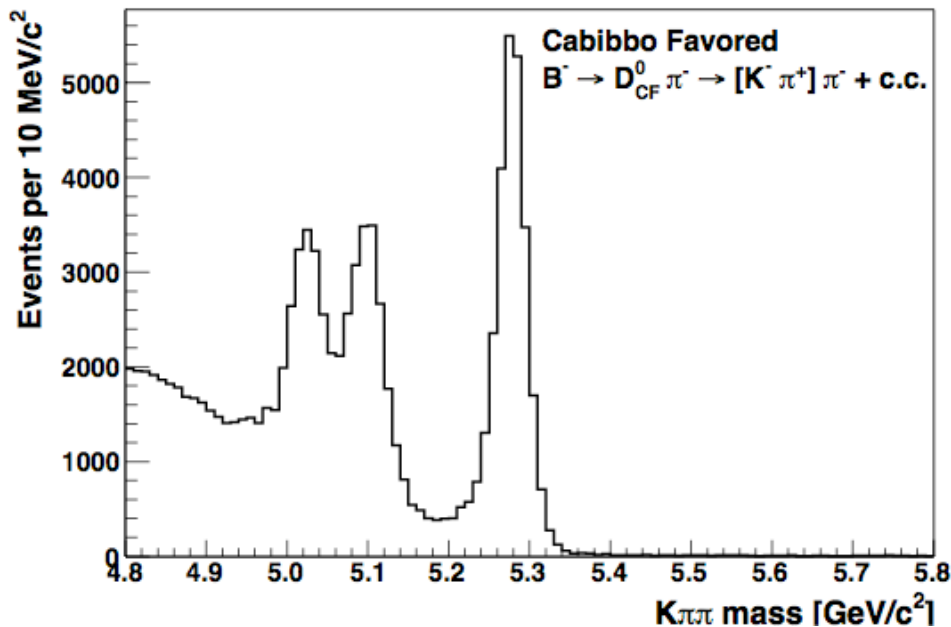


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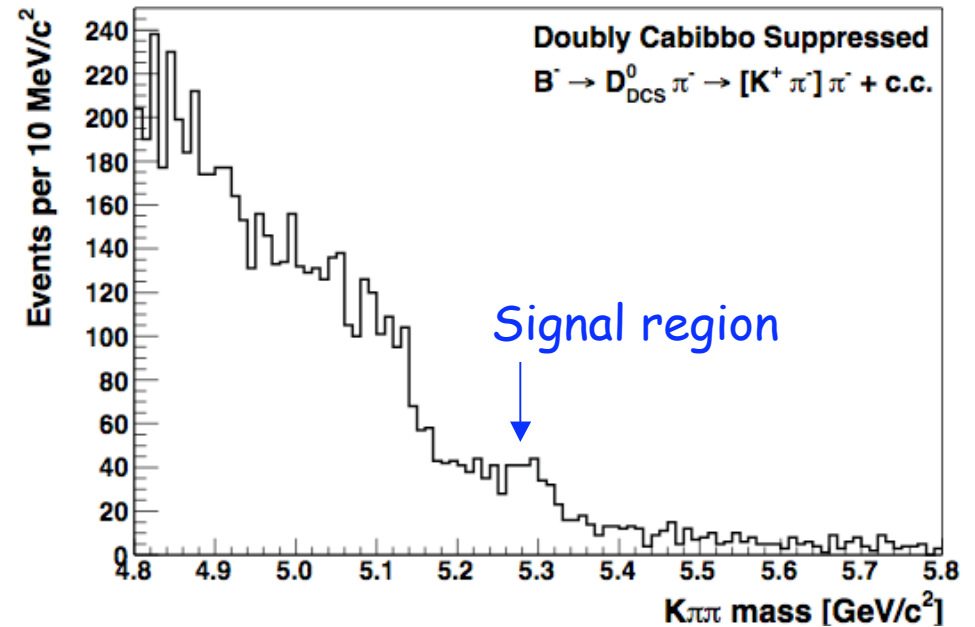
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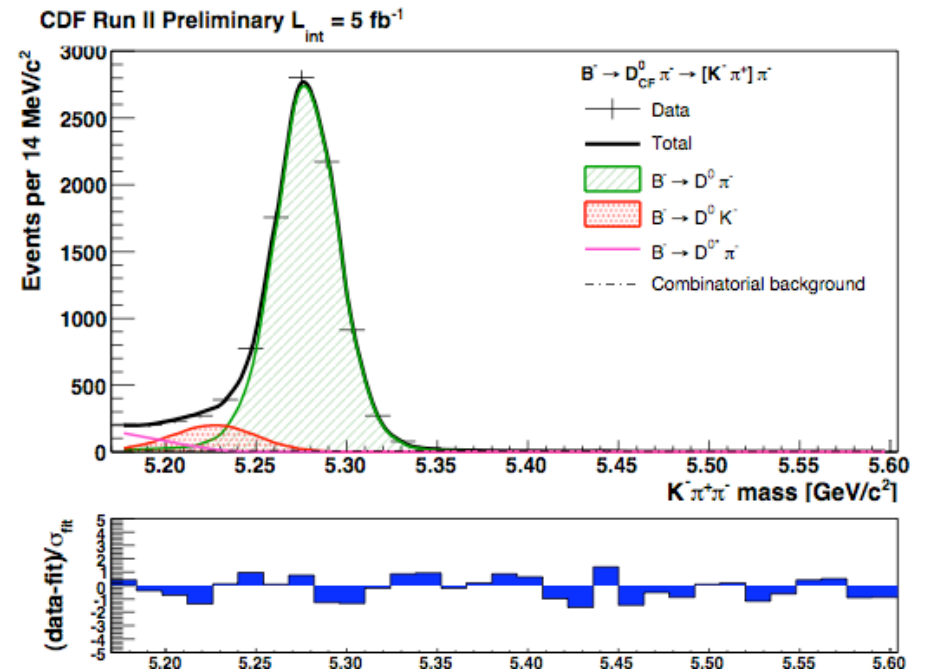
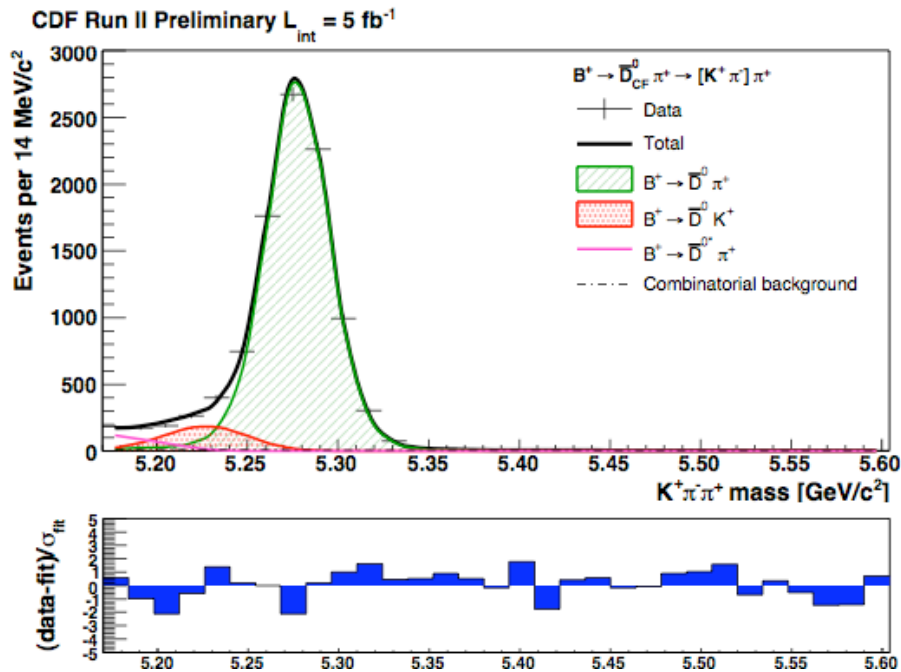
Fit procedure

Use of an *unbinned maximum likelihood fit* (combined on CF and DCS modes) to separate signals contribution using:

- mass information
- particle identification (dE/dx with K- π separation: 1.5σ for $p > 2 \text{ GeV}/c$)

$$B^+ \rightarrow \bar{D}_{CF}^0 \pi^+ \rightarrow [K^- \pi^+] \pi^+$$

$$B^- \rightarrow D_{CF}^0 \pi^- \rightarrow [K^+ \pi^-] \pi^-$$

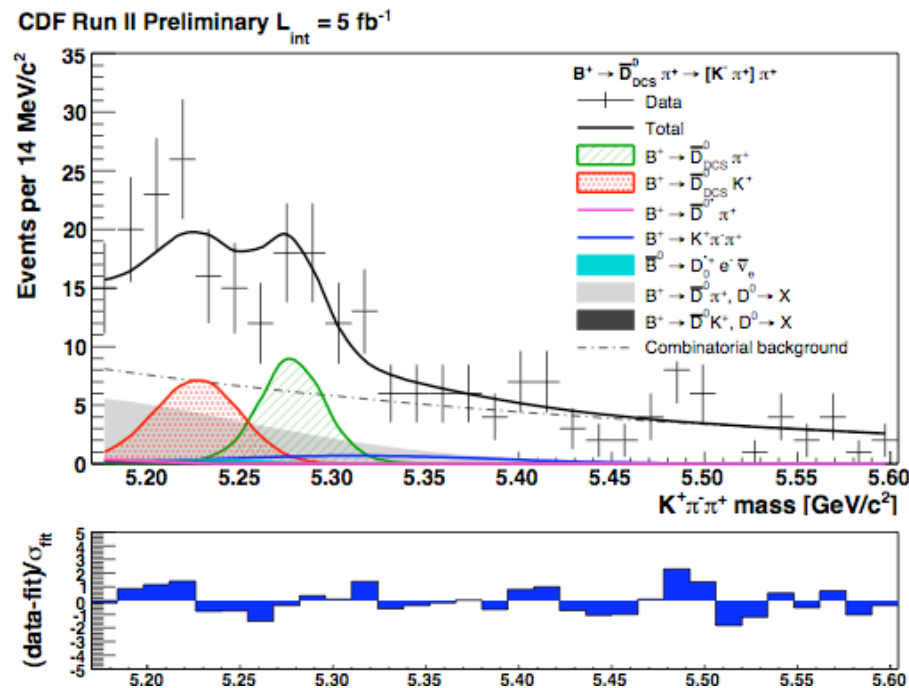


$$\text{Yield } (B \rightarrow D_{CF} K) = 1513 \pm 68 \text{ (5 fb}^{-1}\text{)}$$

$$\text{Yield } (B \rightarrow D_{CF} \pi) = 17677 \pm 146 \text{ (5 fb}^{-1}\text{)}$$

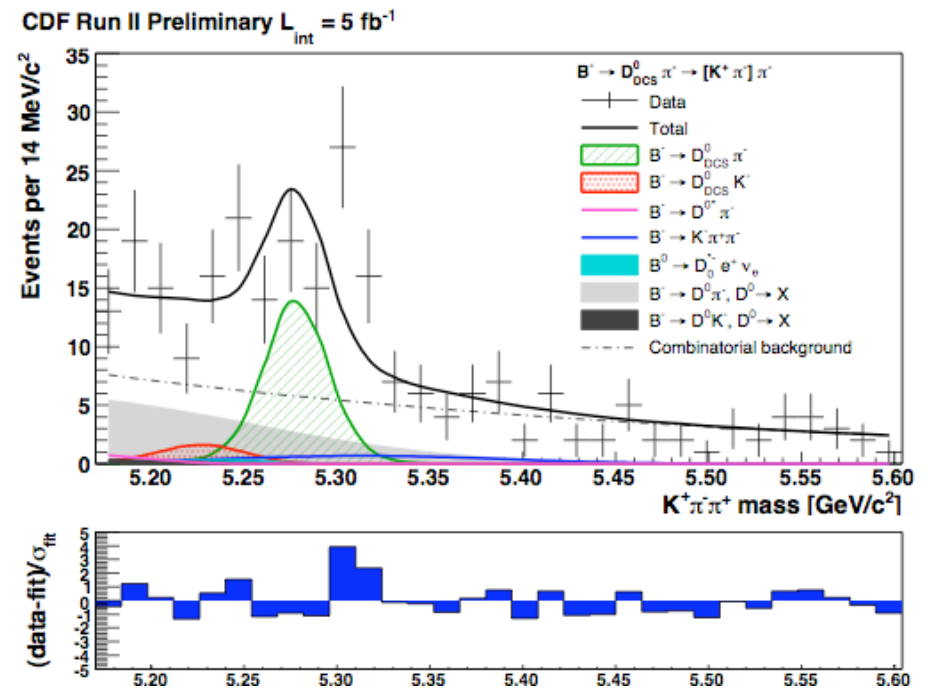
First reconstruction of DCS signals at a hadron collider.

$$B^+ \rightarrow \bar{D}_{DCS}^0 \pi^+ \rightarrow [K^- \pi^+] \pi^+$$



Yield ($B \rightarrow D_{DCS} K$) = 34 ± 14 (5 fb^{-1})
 Yield ($B \rightarrow D_{DCS} \pi$) = 73 ± 16 (5 fb^{-1})

$$B^- \rightarrow D_{DCS}^0 \pi^- \rightarrow [K^+ \pi^-] \pi^-$$



Significance for all DCS
 signals ($D_{DCS} \pi + D_{DCS} K$) $> 5 \sigma$

Results: physics background

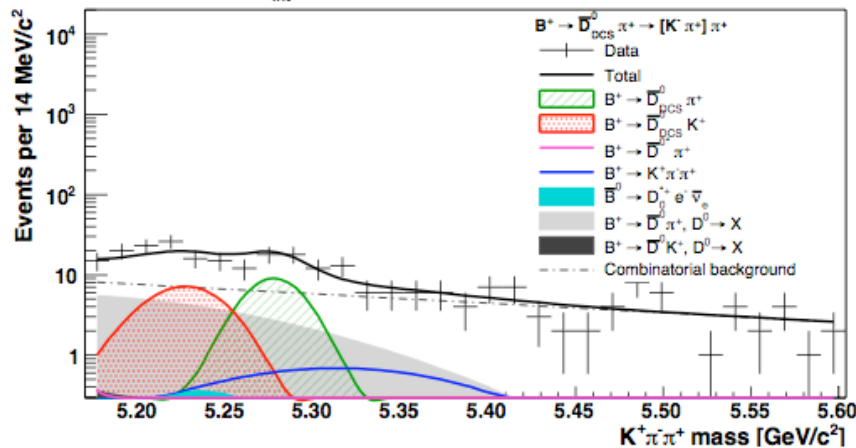
Physics background for DCS:

Decay	Yield
$B^- \rightarrow D^{0*} \pi^-, D^{0*} \rightarrow D^0 \gamma / \pi^0$	3 ± 3
$B^- \rightarrow D^0 \pi^-, D^0 \rightarrow X$	90 ± 13
$B^- \rightarrow D^0 K^-, D^0 \rightarrow X$	4 ± 3
$B^- \rightarrow K^- \pi^+ \pi^-$	18 ± 4
$B^0 \rightarrow D_0^{*-} e^+ \nu_e$	4 ± 3

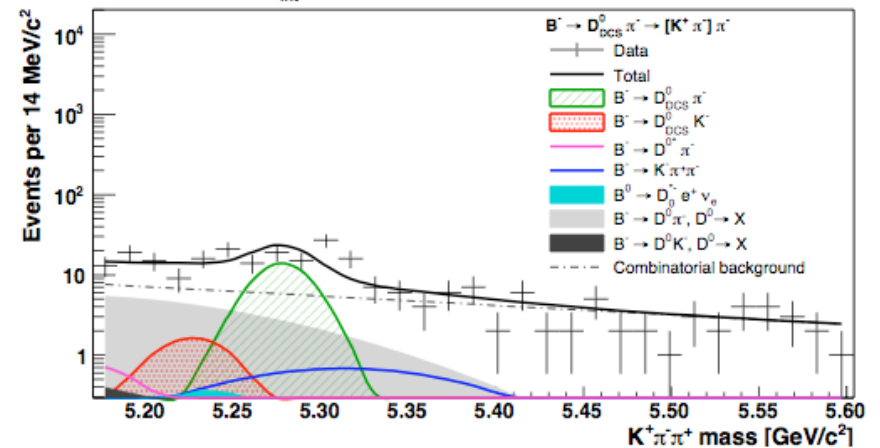
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CDF Run II Preliminary $L_{\text{int}} = 5 \text{ fb}^{-1}$



CDF Run II Preliminary $L_{\text{int}} = 5 \text{ fb}^{-1}$





Results: the observables



First measurement of A_{ADS} and R_{ADS}
at a hadron collider.

$$R_{ADS}(\pi) = 0.0041 \pm 0.0008(stat) \pm 0.0004(syst)$$

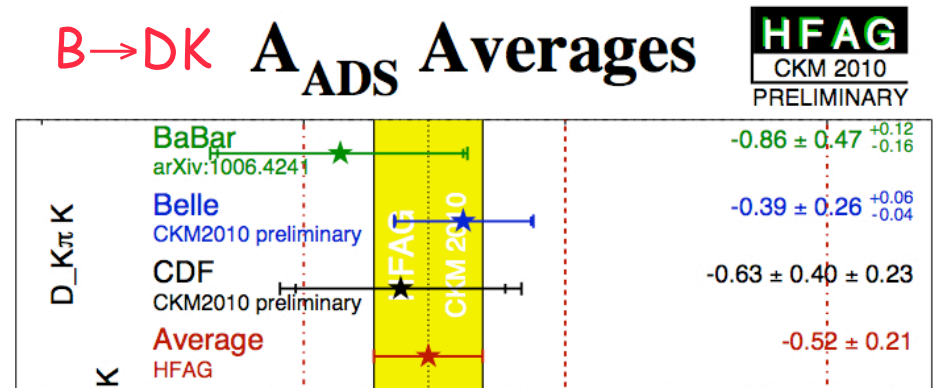
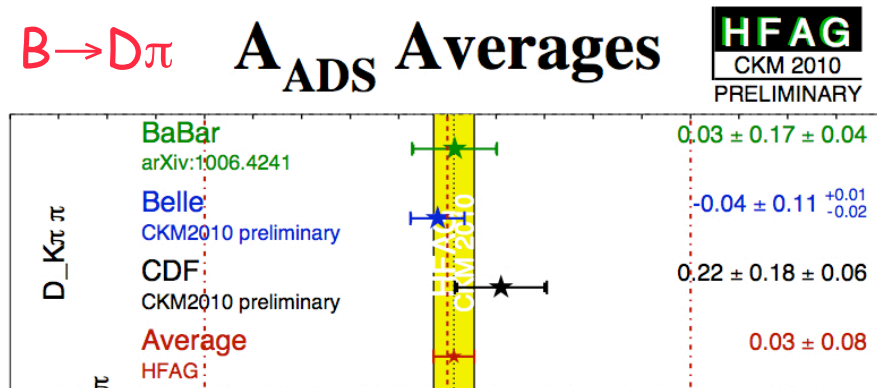
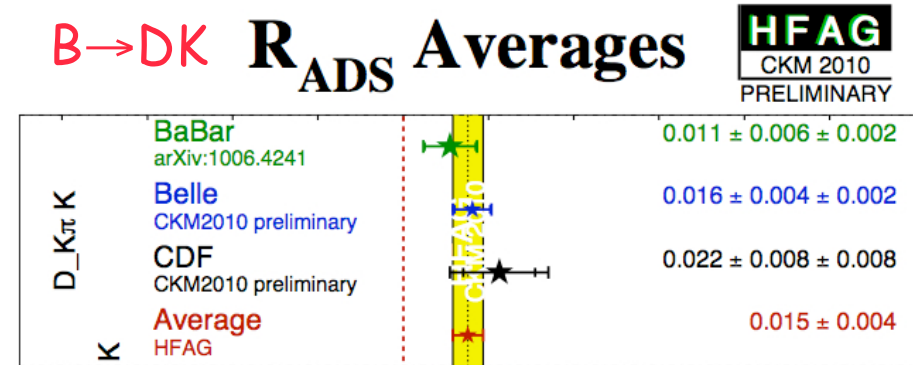
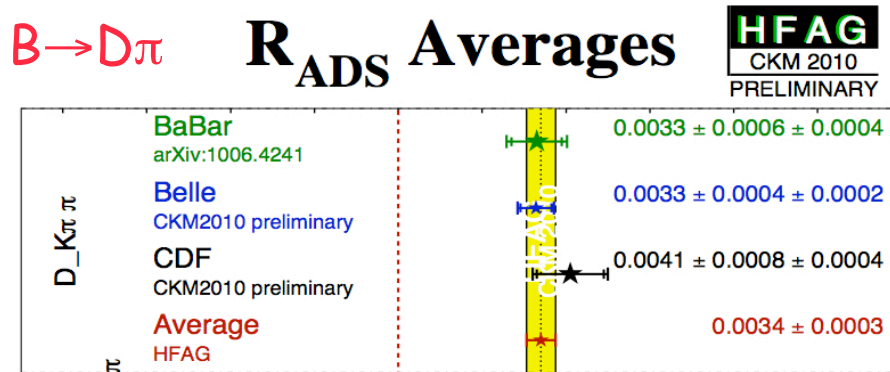
$$A_{ADS}(\pi) = 0.22 \pm 0.18(stat) \pm 0.06(syst)$$

$$R_{ADS}(K) = 0.0225 \pm 0.0084(stat) \pm 0.0079(syst)$$

$$A_{ADS}(K) = -0.63 \pm 0.40(stat) \pm 0.23(syst)$$

(CDF public note 10309)

The results are in agreement and compatible with B-factories





CDF program on γ



The ADS measurement belongs to a *broader program of CDF for measuring γ from trees.*

Recently published the **GLW measurement** using 1 fb^{-1} of data
(*Phys.Rev.D81:031105,2010*)

The GLW method

- Direct CP violation in $B \rightarrow D_{CP} K$ modes
($D_{CP+} \rightarrow \pi^+ \pi^-, K^+ K^-$ and $D_{CP-} \rightarrow K_s^0 \pi^0, K_s^0 \omega, K_s^0 \phi$.)
- very clean method
- small asymmetry, sensitivity to γ proportional to r_B

The observables

$$R_{CP\pm} = \frac{\Gamma(B^- \rightarrow D_{CP\pm}^0 K^-) + \Gamma(B^+ \rightarrow D_{CP\pm}^0 K^+)}{[\Gamma(B^- \rightarrow D^0 K^-) + \Gamma(B^+ \rightarrow D^0 K^+)]/2}$$

$$A_{CP\pm} = \frac{\Gamma(B^- \rightarrow D_{CP\pm}^0 K^-) - \Gamma(B^+ \rightarrow D_{CP\pm}^0 K^+)}{\Gamma(B^- \rightarrow D_{CP\pm}^0 K^-) + \Gamma(B^+ \rightarrow D_{CP\pm}^0 K^+)}$$

From theory:

$$R_{CP\pm} = 1 + r_B^2 \pm 2r_B \cos\delta_B \cos\gamma$$

$$A_{CP\pm} = 2r_B \sin\delta_B \sin\gamma / R_{CP\pm}$$

3 independent equations

$$(A_{CP+} R_{CP+} = -A_{CP-} R_{CP-})_{25}$$

and **3 unknowns** (r_B, γ, δ_B)

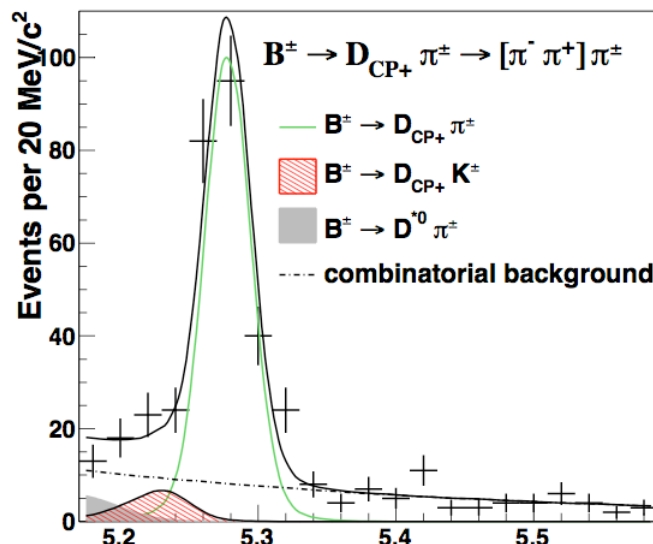
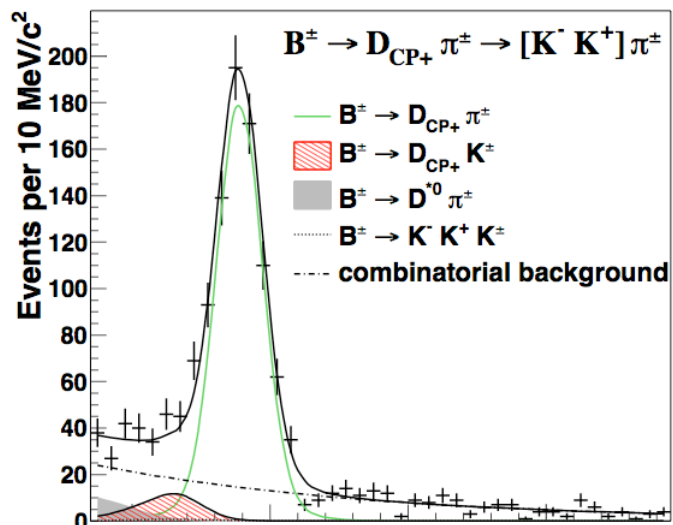


Results



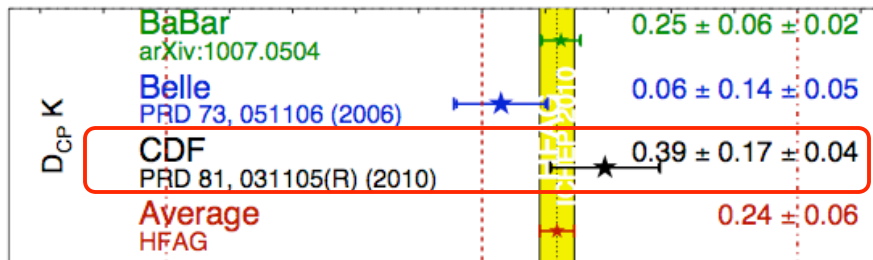
	D mode $B^+ \rightarrow D\pi^+$	$B^- \rightarrow D\pi^-$	$B^+ \rightarrow DK^+$	$B^- \rightarrow DK^-$
$K^- \pi^+$	3769 ± 68	3763 ± 68	250 ± 26	266 ± 27
$K^+ K^-$	381 ± 25	399 ± 26	22 ± 8	49 ± 11
$\pi^+ \pi^-$	101 ± 13	117 ± 14	6 ± 6	14 ± 6

Yield ($B \rightarrow D_{CP+} K$) ~ 90 (1 fb^{-1})



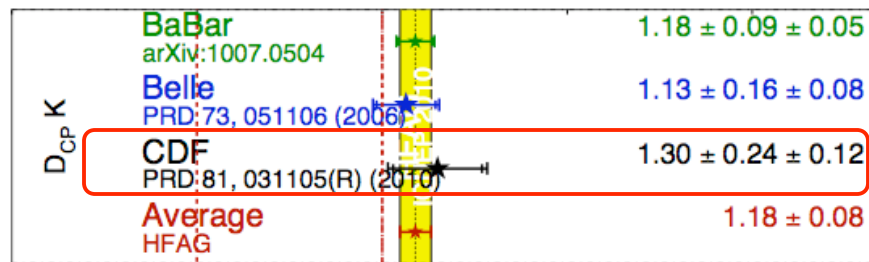
$B \rightarrow DK$ A_{CP+} Averages

HFAG
ICHEP 2010
PRELIMINARY



$B \rightarrow DK$ R_{CP+} Averages

HFAG
ICHEP 2010
PRELIMINARY





Conclusions



- CDF performed:
 - first measurement of A_{ADS} and R_{ADS} at a hadron collider using 5 fb^{-1} .
 - Significance of DCS signals ($D_{DCS} \pi + D_{DCS} K$) $> 5\sigma$
 - first measurement of A_{CP+} and R_{CP+} at a hadron collider using 1 fb^{-1} .
- Demonstrated **capability** of hadron collider with **B to charm decays**. The results are competitive with B-factories
- While B-factories updated the measurement to all dataset available, CDF expect to **double the data-set** by 2011 and give significant updates soon.



BACK-UP



From an LHCb study: resolution on γ (assuming 2fb^{-1})



B mode	D mode	Method	$\sigma(\gamma)$
$B^+ \rightarrow DK^+$	$K\pi + KK/\pi\pi + K\pi\pi\pi$	ADS+GLW	$5^\circ\text{-}15^\circ$
$B^+ \rightarrow D^*K^+$	$K\pi$	ADS+GLW	under study
$B^+ \rightarrow DK^+$	$K_S\pi\pi$	Dalitz	15°
$B^+ \rightarrow DK^+$	$KK\pi\pi$	4-body "Dalitz"	15°
$B^+ \rightarrow DK^+$	$K\pi\pi\pi$	4-body "Dalitz"	under study
$B^0 \rightarrow DK^{*0}$	$K\pi + KK + \pi\pi$	ADS+GLW	$7^\circ\text{-}10^\circ$
$B^0 \rightarrow DK^{*0}$	$K_S\pi\pi$	Dalitz	under study
$B_S \rightarrow D_S K$	$KK\pi$	tagged, $A(t)$	13°
$B \rightarrow \pi\pi, KK$			$4^\circ\text{-}10^\circ$

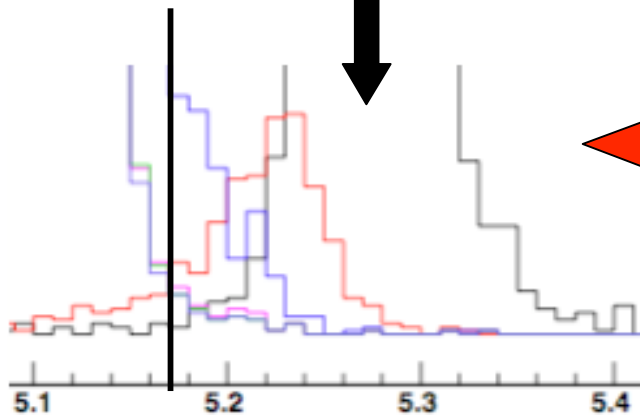
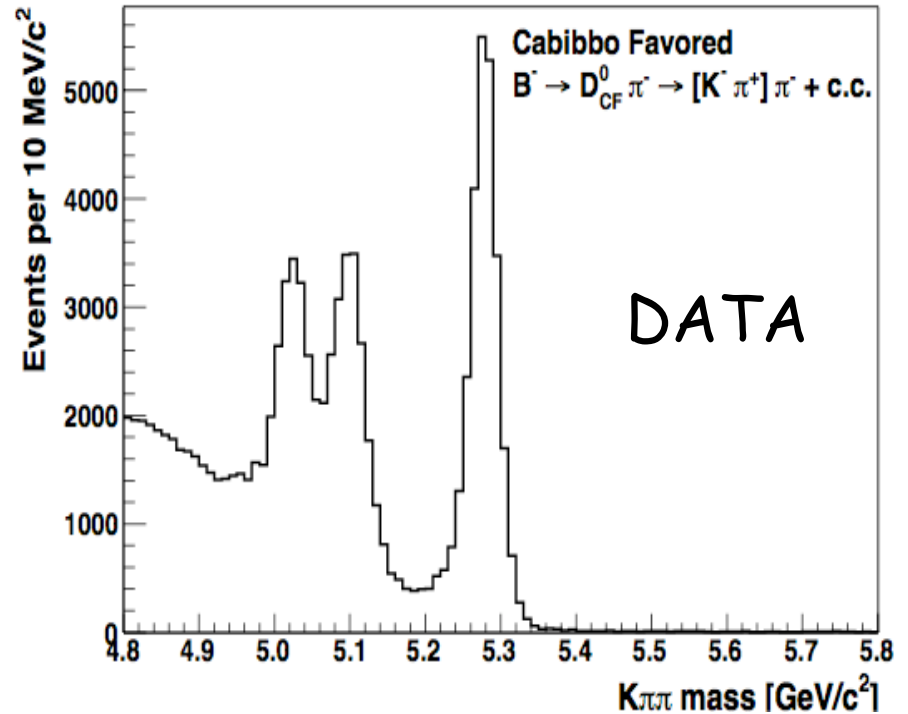
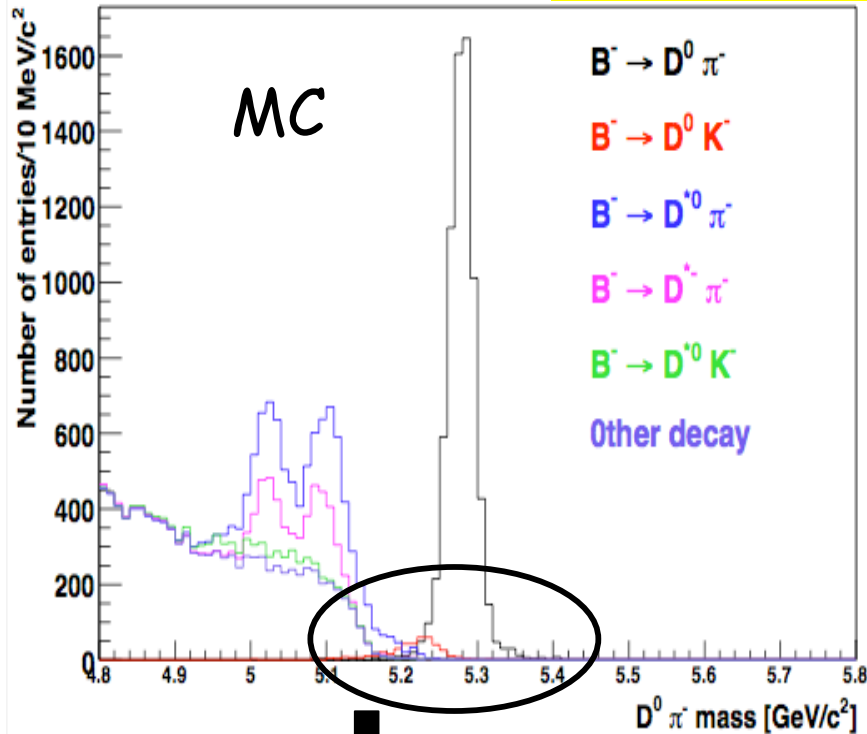
Signal only, no accept. effect

When statistics becomes sufficiently large to actually **see** the **B- \rightarrow DK DCS** decay, the most sensitive method for measuring gamma becomes **ADS** (+GLW), not Dalitz.

Separating DK from other modes

CDF Run II MC

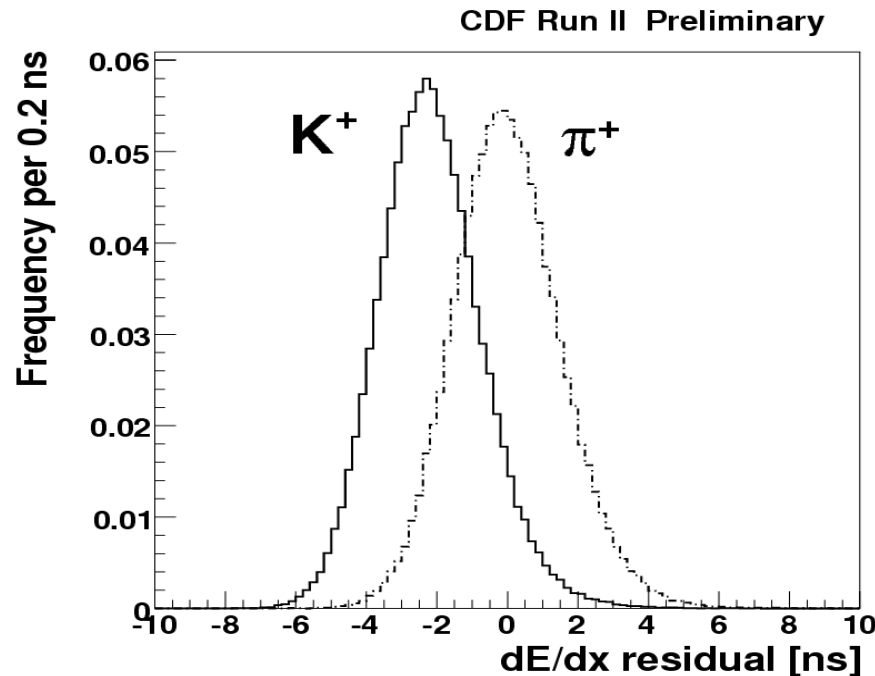
$$B^- \rightarrow D^0_{CF} \pi^- \rightarrow [K^- \pi^+] \pi^- \quad y L_{int} = 5 \text{ fb}^{-1}$$



To reject most of the physical backgrounds, narrow fit windows [5.17, 6.5]

The only significant physics backgrounds are $B^- \rightarrow D^0 \pi^-$ and $B^- \rightarrow D^{*0} \pi^-$

Implementation of a Likelihood FIT using **masses** and **particle identification** (dE/dx) information to determine the signal composition



$K - \pi$ separation: **1.5 σ** for
 $p > 2 \text{ GeV}/c$



ADS: Systematics



Source	$R_{ADS}(\pi)$	$R_{ADS}(K)$	$A_{ADS}(\pi)$	$A_{ADS}(K)$
dE/dx	0.0001	0.0050	0.0560	0.070
combinatorial background	0.0003	0.0037	0.0073	0.153
$B^- \rightarrow [X]_D \pi^-$ shape	0.0002	0.0025	0.0067	0.057
$B^- \rightarrow [X]_D K^-$ shape	-	0.0001	0.0003	0.003
$B^- \rightarrow K^- \pi^+ \pi^-$ shape	0.0001	0.0004	0.0049	0.009
$B^0 \rightarrow D_0^{*-} e^+ \nu_e$ shape	-	0.0003	0.0020	0.007
$B^- \rightarrow D^{*0} \pi^-$ shape	-	0.0005	0.0009	0.013
efficiency	-	0.0001	-	0.003
bias	0.0001	0.0042	0.0159	0.148
Total	0.0004	0.0079	0.059	0.232



ADS: Likelihood



$$\mathcal{L} = \mathcal{L}_{CF+} \cdot \mathcal{L}_{CF-} \cdot \mathcal{L}_{DCS+} \cdot \mathcal{L}_{DCS-}$$

$$\mathcal{L}_{CF+} = \prod_i^{N_{events}} \left[(1 - b_{CF+}) \cdot \left(f_{\pi}^{CF+} \cdot pdf_{\pi}(M, ID) + c^+ \cdot f_{\pi}^{CF+} \cdot pdf_{D^*}(M, ID) + \right. \right. \\ \left. \left. + \left(1 - f_{\pi}^{CF+} - c^+ \cdot f_{\pi}^{CF+} \right) \cdot pdf_K(M, ID) \right) + b_{CF+} \cdot pdf_{comb}(M, ID) \right]$$

$$\mathcal{L}_{DCS+} = \prod_i^{N_{events}} \left[(1 - b_{DCS+}) \cdot \left(f_{\pi}^{DCS+} \cdot pdf_{\pi}(M, ID) + c^+ \cdot f_{\pi}^{DCS+} \cdot pdf_{D^*}(M, ID) + \right. \right. \\ \left. \left. + \left(1 - f_{\pi}^{DCS+} - c^+ \cdot f_{\pi}^{DCS+} \right) \cdot pdf_K(M, ID) \right) + \right. \\ \left. + b_{DCS+} \cdot \left(f_{[X]\pi}^+ \cdot pdf_{[X]\pi}(M, ID) + f_{[X]K}^+ \cdot pdf_{[X]K} + f_{K\pi\pi}^+ \cdot pdf_{K\pi\pi}(M, ID) + \right. \right. \\ \left. \left. f_{B^0}^+ \cdot pdf_{B^0}(M, ID) + (1 - f_{[X]\pi}^+ - f_{[X]K}^+ - f_{K\pi\pi}^+ - f_{B^0}^+) \cdot pdf_{comb}(M, ID) \right) \right]$$

Analogous expressions for negative charges

- $pdf_i(M, ID) = pdf_i(M) \cdot pdf_i(ID)$

- **Fitted parameters**

- $b_{CF, DCS}$ = background fraction for CF and DCS
- $f_{\pi, CF, DCS}$ = $B \rightarrow D^0 \pi$ fraction for CF and DCS signal
- $c = f_{D^*} / f_{\pi}$ (equal for CF and DCS)
- $f_{[X]\pi}$ = fraction of $B \rightarrow D^0 \pi$, $D^0 \rightarrow X$ in DCS reconstruction (constrained from MC)
- $f_{[X]K}$ = fraction of $B \rightarrow D^0 K$, $D^0 \rightarrow X$ in DCS reconstruction (constrained from MC)
- $f_{K\pi\pi}$ = fraction of $B \rightarrow K^- \pi^+ \pi^-$ in DCS reconstruction (constrained from MC)
- f_{B^0} = fraction of $B^0 \rightarrow D^* e \nu$ in DCS reconstruction (constrained from MC)